Entanglement dynamics of two-qubit system in different types of noisy channels *

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Abstract

In this paper, we study entanglement dynamics of a two-qubit extended Werner-like state locally interacting with independent noisy channels, i.e., amplitude damping, phase damping and depolarizing channels. We show that the purity of initial entangled state has direct impacts on the entanglement robustness in each noisy channel. That is, if the initial entangled state is prepared in mixed instead of pure form, the state may exhibit entanglement sudden death (ESD) and/or be decreased for the critical probability at which the entanglement disappear.

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I. INTRODUCTION

In the last two decades, entanglement attracts much attention due to the powerful applications in quantum information process and quantum computing.[1,2] In order to realize quantum information process, great effort has been devoted to studying and characterizing the entanglement in cavity QED [3−5] and spin systems[6,7] schemes. Therefore, it is of increasingly importance to understand entanglement behaviors of quantum system in realistic situations, where the system unavoidably loses its coherence due to interactions with the environment. In this context, a peculiar dynamical feature of entanglement was discovered that the entanglement can vanish completely in a finite time, in striking contrast with decoherence of its individual constituent that decays only asymptotically. Such a surprising phenomenon was termed entanglement sudden death (ESD)[8]. Because of its intrinsic and practical interests, ESD has attracted much attention in theory[9−14] and confirmed experimentally[15] for the case of two qubits. The ESD phenomenon illustrates the fact that the global behavior of an entangled system may be markedly different from the individual and local behavior of its constituents.[14]

From a practical point of view, an entangled state undergoing ESD would put a limitation on the time of its application in practice since it is less robust than one without ESD. Hence, to find various conditions under which the ESD occurs seems to be very necessary. It has shown that ESD is sensitive to the type of initial entanglement, i.e., depending on the type of initial state, the entanglement may or may not exhibit ESD.[10] Besides the initial condition of an entangled state, the environment is another decided factor responsible for the dynamical behaviors of entanglement. In Ref. [14], the authors studied entanglement dynamics in three types of noisy channels. It was shown that for the same entangled states, their entanglement may exhibit completely different behaviors involving the appearance of ESD in different types of environments. However, they only considered the pure case of initial entangled states. In this paper, we shall study the entanglement dynamics of an entangled state subject to various noisy channels by paying more addition to the initial condition of the state. We show that the purity of initial entangled state is closely related to the entanglement robustness in each noisy channel. That is, if the initial entangled state is prepared in mixed instead of pure form, the state may exhibit ESD and/or be decreased for the critical probability at which the entanglement disappear.

To quantify the entanglement of a two-qubit system we adopt Wootters' concurrence[16]. The

concurrence $C(\rho_{AB})$ for the density matrix ρ_{AB} of a two-qubit system AB is defined as

$$C(\rho_{AB}) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \tag{1}$$

where λ_i are the eigenvalues of the matrix $\zeta = \rho(\sigma_y^A \otimes \sigma_y^B)\rho^*(\sigma_y^A \otimes \sigma_y^B)$ arranged in decreasing order. Here $\sigma_y^{A(B)}$ are the y-Pauli matrix acting on qubit A (B) and ρ^* is the complex conjugation of ρ in the standard (computational) basis. For the separate state $C = 0$ whereas $C = 1$ for maximally entangled state.

For the initial state of qubit-pair AB, instead of Bell-like and Werner states[17], we shall consider the following extended Werner-like state

$$\rho_{AB}(0) = r \left|\Phi\right\rangle_{ABAB} \left\langle\Phi\right| + \frac{1-r}{4} I_{AB}, \tag{2}$$

with r the purity of the initial state of qubits AB, I_{AB} the 4×4 identity matrix and

$$\left|\Phi\right\rangle_{AB} = \left(\sin\theta \left|00\right\rangle + \cos\theta \left|11\right\rangle\right)_{AB}, \tag{3}$$

the Bell-like state. Obviously, the state in Eq. (2) reduces to the standard Werner state when $\theta = \pi/4$ and to Bell-like pure state when $r = 1$. By dealing with the above extended Werner-like state, we are able to study the effect of mixedness of the initial entangled state. Both the Bell-like state and Werner state, and so the extended Werner-like state, belong to the so-called X-class state[12] whose density matrix is of the form

$$\rho_{AB} = \begin{pmatrix} x & 0 & 0 & v \\ 0 & y & u & 0 \\ 0 & u^* & z & 0 \\ v^* & 0 & 0 & w \end{pmatrix}, \tag{4}$$

with x, y, z, w real positive and u, v complex quantities. The X-class states have the property that the corresponding two-qubit density matrix preserves the X-form during the system evolution. For the X-state (4), the concurrence can be derived as

$$C(\rho_{AB}) = 2\max\{0, |u| - \sqrt{xw}, |v| - \sqrt{yz}\}. \tag{5}$$

We consider three paradigmatic types of noisy channels, i.e., amplitude damping channel, phase damping channel and depolarization channel. Each one of the two qubits A and B individually coupled to its own noisy environment, implying no any interaction, direct or indirect, between

them. The dynamics of each qubit is governed by a master equation that gives rise to a completely positive trace-preserving map $\mathcal{E}_i$ (with $i = A, B$) describing the evolution as $\rho_i = \mathcal{E}_i \rho_i(0)$, where $\rho_i(0)$ and ρ_i are, respectively, the initial and evolved reduced states of the i-th subsystem. In the following, we shall consider the entanglement dynamics of the state (2) in the three noisy channels, respectively.

II. AMPLITUDE DAMPING CHANNEL

The first noisy channel we consider is the amplitude damping (AD) channel which can be represented via the Kraus representation as[1,14]

$$\mathcal{E}_i^{AD} \rho_i = E_0 \rho_i E_0^\dagger + E_1 \rho_i E_1^\dagger, \tag{6}$$

with $E_0 = |0\rangle\langle 0| + \sqrt{1-p}\,|1\rangle\langle 1|$ and $E_1 = \sqrt{p}\,|0\rangle\langle 1|$ being its Kraus operators. $p \equiv p(t) \equiv 1 - e^{-\frac{1}{2}\gamma t}$ is the probability of the qubit exchanging a quantum with the bath at time t, and γ is the zero-temperature dissipation rate. After time-evolving, the initial state of qubits A, B in Eq. (2) evolves into $\rho_{AB}(p)$ which has the X-form (4) with the matrix elements are given as

$$\begin{aligned}
x &= \frac{1-r}{4}(1+p)^2 + r(\cos^2(\theta)p^2 + \sin^2(\theta)), \\
y &= z = \frac{1-r}{4}(1-p^2) + r\cos^2(\theta)p(1-p), \\
w &= (\frac{1-r}{4} + r\cos^2(\theta))(1-p)^2, \\
v &= r\sin\theta\cos\theta(1-p), \\
u &= 0.
\end{aligned} \tag{7}$$

By virtue of Eq. (5), we can get the concurrence $C(\rho_{AB}(p))$ of $\rho_{AB}(p)$ as

$$C(\rho_{AB}(p)) = 2\max\{0, |v| - \sqrt{yz}\}. \tag{8}$$

If the initial state of qubits A, B is pure, i.e., $r = 1$, from the relation $|v| - \sqrt{yz} = 0$ we can get the critical probability p_c at which the entanglement disappear as $p_c = |\frac{\sin\theta}{\cos\theta}|$. For $\theta < \pi/4$, p_c is always smaller than 1, meaning that the entanglement disappears before the steady state is asymptotically reached.[14] Thus $\theta < \pi/4$ is the condition for the occurrence of ESD when the initial state of A, B is pure, i.e. $r = 1$. The relations between $C(\rho_{AB}(p))$ and p, θ are plotted in FIG. 1 for $r = 1$. However, we shall pay more attention to the case of $r < 1$, i.e., the initial

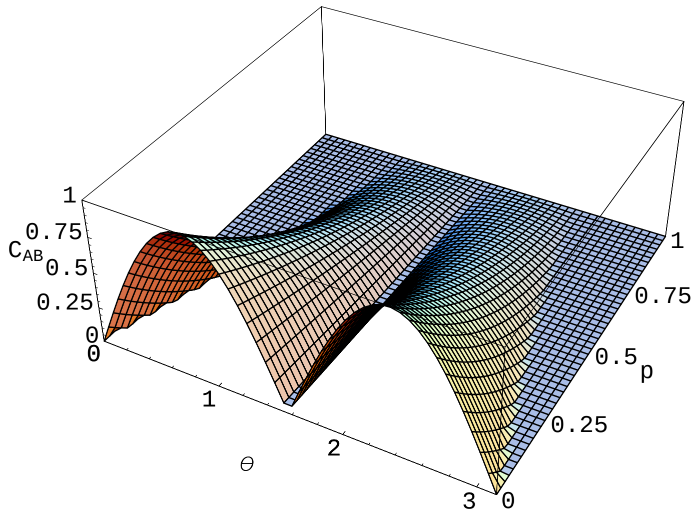

FIG. 1: $C_{AB} \equiv C(\rho_{AB}(p))$ as functions of θ and p for the case of $r = 1$. The ESD appears for $\theta < \pi/4 + k\pi$.

entangled state is prepared mixed form. In this case, the critical probability $p_c = \frac{4r|\sin\theta\cos\theta|+r-1}{4r|\cos^2\theta|-r+1}$, which is related to both the degree and purity of initial entanglement in terms of θ and r. The condition of appearance of ESD is derived as $|\sin\theta\cos\theta| - \cos^2\theta < \frac{1}{2}(\frac{1}{r} - 1)$, from which we can see that the range of θ for appearing ESD depends on the value of r. As an example, for $r = 0.7$ we plot in FIG. 2 the concurrence $C(\rho_{AB}(p))$ as functions of p and θ, where we can see that ESD occurs for all the possible values of θ, in striking contrast with the case of $r = 1$ in FIG. 1. Hence, ESD phenomenon is closely related to the purity of initial entangled state.

III. PHASE DAMPING CHANNEL

In this section, we consider the phase damping channel (PD) in which there is loss of quantum information with probability p, but without any energy exchange. It is defined as[14]

$$\mathcal{E}_i^{PD}\rho_i = (1-p)\rho_i + p(|0\rangle\langle 0|\rho_i|0\rangle\langle 0| + |1\rangle\langle 1|\rho_i|1\rangle\langle 1|). \tag{9}$$

After time-evolving, the diagonal terms of initial state in Eq. (2) of qubits A, B remains the same, whereas the off-diagonal ones are multiplied by $(1-p)^2$. By virtue of Eq. (5), we can get the concurrence $C(\rho_{AB}(p))$ of $\rho_{AB}(p)$ as

$$C(\rho_{AB}(p)) = 2\max\{0, |v| - \sqrt{yz}\}, \tag{10}$$

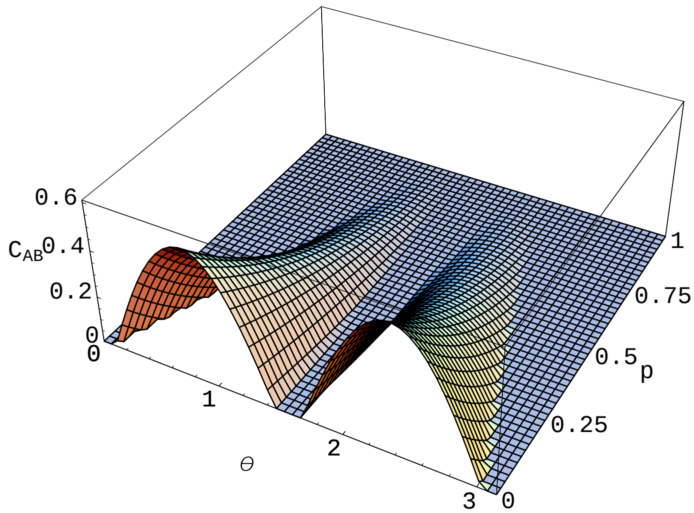

FIG. 2: $C_{AB} \equiv C(\rho_{AB}(p))$ as functions of θ and p for the case of $r = 0.7$. The ESD appears for all the possible values of θ.

with $y = z = \frac{1-r}{4}$ and $v = r \sin\theta \cos\theta (1-p)^2$. Obviously, if the initial state of qubits AB is pure, i.e., $r = 1$, we have $y = z = 0$, thus $C(\rho_{AB}(p)) = 2\sin\theta\cos\theta(1-p)^2 \geq 0$ with equality hold for $p = 1$ implying non-existence of ESD for all the possible values of θ. However, if the initial state of qubits AB is mixed the situation will be different. From the relation $|v| - \sqrt{yz} = 0$ we can get the critical probability p_c as $p_c = 1 - \sqrt{\frac{1-r}{4r|\sin\theta\cos\theta|}}$. For $r < 1$, p_c is always smaller than 1, meaning that in the PD channel the entangled state (2) always suffers ESD if the state is prepared in the mixed form. To make a comparison, in FIG. 3 and FIG. 4, we plot $C(\rho_{AB}(p))$ as functions of θ and p for $r = 1$ and $r = 0.7$, respectively. Hence, the close relation between ESD and the purity of initial entangled state can also be obtained in the PD channel.

IV. DEPOLARIZING CHANNEL

Next, we consider entanglement dynamics in the depolarizing (D) channel. The D channel represents the situation where the i-th qubit remains untouched with probability $1-p$, or is depolarized-meaning that its state is taken to the maximally mixed state-with probability p. It can be expressed as[14]

$$\mathcal{E}_i^D \rho_i = (1-p)\rho_i + p\frac{\mathbf{I}}{2}, \tag{11}$$

with $\mathbf{I}$ is the identity operator.

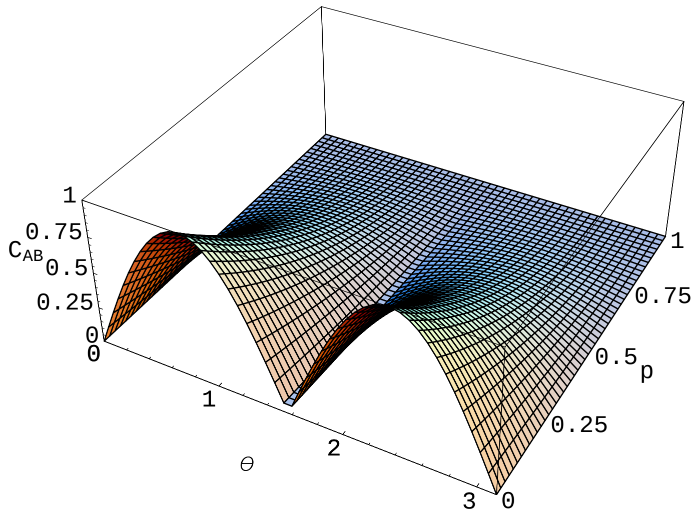

FIG. 3: $C_{AB} \equiv C(\rho_{AB}(p))$ as functions of θ and p for the case of $r = 1$. The ESD does not appear for all the possible values of θ.

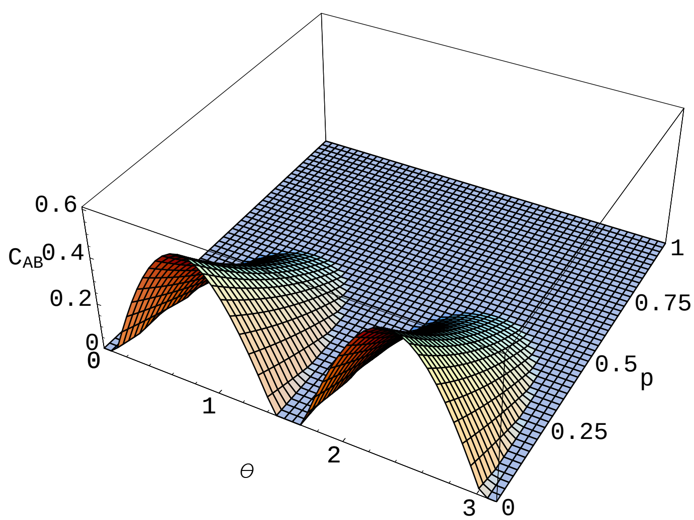

FIG. 4: $C_{AB} \equiv C(\rho_{AB}(p))$ as functions of θ and p for the case of $r = 0.7$. The ESD instead appears for all the possible values of θ.

After time-evolving, the density matrix (2) evolves into $\rho_{AB}(p)$ which remains the X form (4)

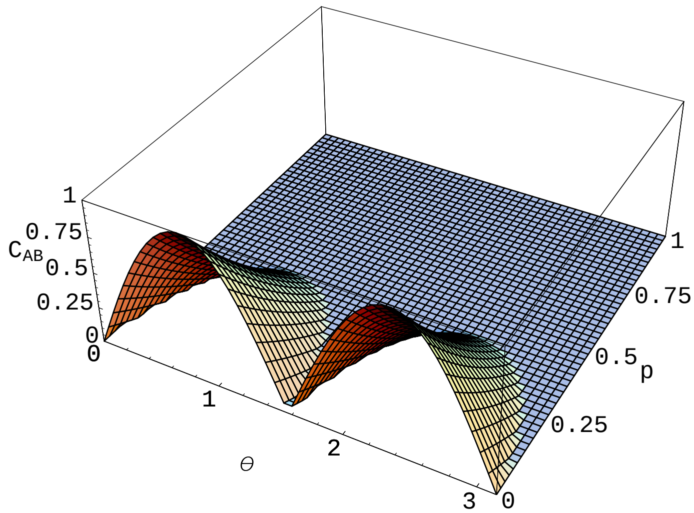

FIG. 5: $C_{AB} \equiv C(\rho_{AB}(p))$ as functions of θ and p for the case of $r = 1$. The ESD happens for all the possible values of θ.

with the matrix elements are given as

$$
\begin{aligned}
x &= (1-\frac{p}{2})^2(\frac{1-r}{4} + r\sin^2\theta) + p(1-\frac{p}{2})\frac{1-r}{4} + (\frac{p}{2})^2(\frac{1-r}{4} + r\cos^2\theta), \\
y &= z = \frac{1-r}{4}(1-p+\frac{p^2}{2}) + \frac{p}{4}(1-\frac{p}{2})(1+r), \\
w &= (1-\frac{p}{2})^2(\frac{1-r}{4} + r\cos^2\theta) + p(1-\frac{p}{2})\frac{1-r}{4} + (\frac{p}{2})^2(\frac{1-r}{4} + r\sin^2\theta), \\
v &= r\sin\theta\cos\theta(1-p)^2, \\
u &= 0.
\end{aligned} \tag{12}
$$

By virtue of Eq. (5), we can get the concurrence $C(\rho_{AB}(p))$ of $\rho_{AB}(p)$ as

$$C(\rho_{AB}(p)) = 2\max\{0, |v| - \sqrt{yz}\}. \tag{13}$$

For $r = 1$ the state (2) always undergoes ESD for all the possible values of θ in the time evolution process as shown in FIG. 5, where we plot $C(\rho_{AB}(p))$ as functions of p and θ. It is known that ESD is a representation for the fragility of entanglement. If we take the mixedness of initial entanglement into account, the robustness of entanglement will be reduced further. In FIG. 6 we plot $C(\rho_{AB}(p))$ as function of p for different r, where we can see that the critical probability will be decreased with r.

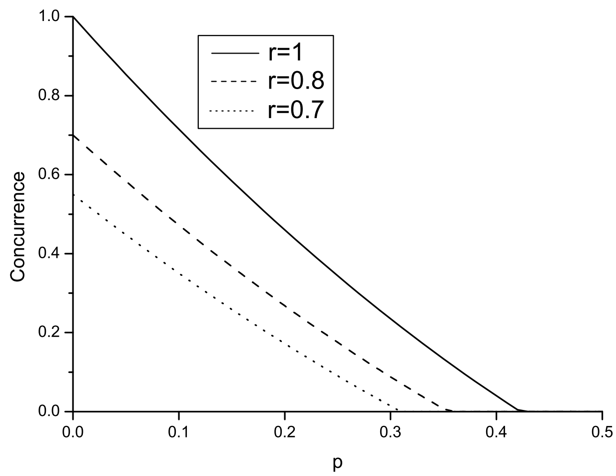

FIG. 6: $C(\rho_{AB}(p))$ as function of p for $\theta = \pi/4$ for various r. The critical probability decreased with r.

V. CONCLUSIONS

In conclusion, we have studied the entanglement evolution of a two-qubit entangled state subject to independent environments, i.e., the amplitude damping, phasing damping and depolarizing channels. The initial entangled state is prepared in an extended Werner-like form (2), thus we can investigate the relations between the purity of the entangled state and its dynamical behaviors. We find that the dynamical behaviors of the two-qubit entanglement are closely related to its purity as well as the types of noisy channels. The mixedness has direct impacts on the robustness of entanglement in the sense that it can result in ESD and/or decrease the critical probability. The understanding of entanglement behaviors in realistic system is a precondition for its application in practice, thus studying the conditions which may influence the entanglement dynamics in various situations prove very important and necessary.

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